CSE-433 2013 Fall - Final Exam

Out: 11:59am, Dec 18 (Wednesday), Due: 11:59am, Dec 19 (Thursday)

1 Admissibility of cut for S'' (75 points)

Consider a sequent calculus S'' for a fragment of propositional logic which has two structural rules for weakening and contraction. We use a sequent of the form $\Gamma \Longrightarrow C$.

$$\frac{}{A \Longrightarrow A}\ \mathit{Init}$$

$$\frac{\Gamma \Longrightarrow C}{\Gamma, A \Longrightarrow C}\ \mathit{Weaken} \qquad \frac{\Gamma, A, A \Longrightarrow C}{\Gamma, A \Longrightarrow C}\ \mathit{Cont}$$

$$\frac{\Gamma_1 \Longrightarrow A \quad \Gamma_2, B \Longrightarrow C}{\Gamma_1, \Gamma_2, A \supset B \Longrightarrow C}\ \supset\! L \qquad \frac{\Gamma, A \Longrightarrow B}{\Gamma \Longrightarrow A \supset B}\ \supset\! R$$

The goal is to prove admissibility of cut for S'':

Theorem 1.1. *If $\Gamma \Longrightarrow A$ and $\Gamma, A \Longrightarrow C$, then $\Gamma \Longrightarrow C$.*

Give a proof of Theorem 1.1. If necessary, introduce lemmas or generalize the statement in Theorem 1.1. Do not, however, exploit admissibility of cut in another sequent calculus for the same fragment of propositional logic, such as the sequent calculus in the course notes.

2 CPS translation based on call-by-name (25 points)

We have studied in class the CPS translation which is based on the call-by-value reduction strategy. To be specific, the translation of $\lambda x\!:\!A.\,M$ and $M\ N$ follows the order of evaluating terms in the call-by-value reduction strategy. Hence, by evaluating M° obtained by the CPS translation of an original term M, we can simulate the call-by-value reduction strategy (regardless of the reduction strategy for evaluating M°).

We wish to give a CPS translation based on the call-by-name reduction strategy. The goal is to figure out x°, $(\lambda x\!:\!A.\,M)^\circ$, and $(M\ N)^\circ$ based on the call-by-name reduction strategy.

We use the following definitions:

$$\begin{array}{rcl} P^\circ & = & P \\ (A \supset B)^\circ & = & \neg\neg A^\circ \supset \neg\neg B^\circ \end{array}$$

$$\begin{array}{rcl} \Gamma^\circ & = & \{x : \neg\neg A^\circ \mid x : A \in \Gamma\} \\ \neg\Delta^\circ & = & \{x : \neg A^\circ \mid x : A \ \mathit{false} \in \Delta\}. \end{array}$$

Theorem 2.1. *If $\Gamma; \Delta \vdash_{\mathsf{K}} M : C$, there exists a proof term M° such that $\Gamma^\circ, \neg\Delta^\circ \vdash_{\mathsf{I}} M^\circ : \neg\neg C^\circ$.*

Find the translation x°, $(\lambda x\!:\!A.\,M)^\circ$, and $(M\ N)^\circ$ so that the above theorem holds.