

Name:

Hemos ID:

CSE-433 Logic in Computer Science 2011 Midterm exam

- This is a closed-book exam. No other material is permitted.
- It consists of 4 problems worth a total of 100 points.
- There are 11 pages in this exam.
- You have one and a half hours for this exam.

	Problem 1	Problem 2	Problem 3	Problem 4	Total
Score					
Max	EC	35	25	40	100

제 1 절 대학생활 [Extracredit]

1.1 전산논리

이번학기 전산논리 과목을 한 단어로 요약하면 무엇인가요?

제 2 절 Propositional logic [35pts]

Question 1. [5 pts] Write the introduction and elimination rules for disjunction \vee using truth judgments of the form $C \text{ true}$.

Question 2. [5 pts] Use the judgments in Question 1 and show that local soundness holds for disjunction \vee .

Question 3. [5 pts] Use the inference rules in Question 1 and show that local completeness holds for disjunction \vee .

Question 4. [5 pts] Write the introduction and elimination rules for disjunction \vee using hypothetical judgments of the form $\Gamma \vdash C \text{ true}$ where Γ is a collection of antecedents.

Question 5. [5 pts] Use the hypothetical judgements in Question 4 and state the two structural properties weakening and contraction.

Question 6. [5 pts] Write a proposition that is logically equivalent to $(A \vee B) \supset C$. Your answer should not express trivial logical equivalence based on commutativity, idempotence, or properties of truth and falsehood. Here are examples of bad answers: $(B \vee A) \supset C$, $(A \vee (\top \wedge B)) \supset C$, and $(A \vee (\perp \vee B)) \supset C$.

Question 7. [5 pts] Suppose that your answer to the previous question is a proposition D . Give a proof of $((A \vee B) \supset C) \supset D$ *true* in the natural deduction style.

제 3 절 First-order logic [25pts]

Question 1. [5 pts] Write the introduction and elimination rules for the existential quantification using truth judgments of the form $C \text{ true}$.

Question 2. [5 pts] Use the inference rules in Question 1 and show that local soundness holds for the existential quantification.

Question 3. [5 pts] Give the typing rules for proof terms for the existential quantification.

Question 4. [10 pts] Assume the following typing rules for characterizing natural numbers:

$$\begin{array}{c}
 \overline{\text{Nat}_0 : \text{Nat}(\mathbf{0})} \text{ Zero} \quad \overline{\text{Nat}_s : \forall x. \text{Nat}(x) \supset \text{Nat}(\mathbf{s}(x))} \text{ Succ} \\
 \\
 \overline{\text{Eq}_i : \forall x. \text{Eq}(x, x)} \text{ Eq}_i \quad \overline{\text{Eq}_t : \forall x. \forall y. \forall z. (\text{Eq}(x, y) \wedge \text{Eq}(x, z)) \supset \text{Eq}(y, z)} \text{ Eq}_t \\
 \\
 \overline{\text{Lt}_s : \forall x. \text{Lt}(x, \mathbf{s}(x))} \text{ Lt}_s \quad \overline{\text{Lt}_{\neg} : \forall x. \forall y. \text{Eq}(x, y) \supset \neg \text{Lt}(x, y)} \text{ Lt}_{\neg}
 \end{array}$$

Give a proof term of type $\neg \exists x. \text{Eq}(x, \mathbf{0}) \wedge \text{Eq}(x, \mathbf{s}(\mathbf{0}))$.

제 4 절 Datatypes [40pts]

Question 1. [5 pts] The introduction rules for datatype `nat` are:

$$\frac{}{\mathbf{0} \in \text{nat}} \text{natI}_0 \quad \frac{t \in \text{nat}}{\mathbf{s}(t) \in \text{nat}} \text{natI}_s$$

We use $\text{ind } u(t)$ of $u(\mathbf{0}) \Rightarrow M \mid u(\mathbf{s}(x)) \Rightarrow N$ for the proof term corresponding to the elimination rule based on induction on terms. Give its typing rule.

Question 2. [10 pts] We abbreviate $EQ(m, n)$ as $m =_{\mathbf{N}} n$ where both m and n belong to datatype `nat`. We use the following proof terms for the introduction rules for $m =_{\mathbf{N}} n$.

$$\frac{}{\text{eqI}_0 : \mathbf{0} =_{\mathbf{N}} \mathbf{0}} =_{\mathbf{N}}\text{I}_0 \quad \frac{M : m =_{\mathbf{N}} n}{\text{eqI}_s(M) : \mathbf{s}(m) =_{\mathbf{N}} \mathbf{s}(n)} =_{\mathbf{N}}\text{I}_s$$

Design three proof terms for the elimination rules for $m =_{\mathbf{N}} n$ which are obtained by inverting the above two introduction rules, and give their typing rules. (Writing only the typing rules is okay.)

Question 3. [5 pts] Give a proof term $eqNat$ of type $\forall x \in \text{nat}. x =_{\mathbb{N}} x$.

Question 4. [10 pts] Give a proof term *trans* of type $\forall x \in \text{nat}. \forall y \in \text{nat}. \forall z \in \text{nat}. x =_{\mathbb{N}} y \supset y =_{\mathbb{N}} z \supset x =_{\mathbb{N}} z$.

Question 5. [10 pts] Assume the following definition of *plus*:

$$plus \in \text{nat} \rightarrow \text{nat} \rightarrow \text{nat}$$

$$plus = \lambda x \in \text{nat}. \lambda y \in \text{nat}. \text{rec } p(x) \text{ of } p(\mathbf{0}) \Rightarrow y \mid p(\mathbf{s}(z)) \Rightarrow \mathbf{s}(p(z))$$

Using definitional equality, give a proof term *comp* of type $\forall x \in \text{nat}. \forall y \in \text{nat}. x + \mathbf{s}(y) =_{\mathbf{N}} \mathbf{s}(x + y)$. You may use *eqNat* given in Question 3.