CSE-433 Logic in Computer Science 2011
Midterm exam

• This is a closed-book exam. No other material is permitted.
• It consists of 4 problems worth a total of 100 points.
• There are 11 pages in this exam.
• You have one and a half hours for this exam.

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제 2 절  Propositional logic [35pts]

Question 1. [5 pts] Write the introduction and elimination rules for disjunction $\lor$ using truth judgments of the form $C\ true$.

Question 2. [5 pts] Use the judgments in Question 1 and show that local soundness holds for disjunction $\lor$.
**Question 3. [5 pts]** Use the inference rules in Question 1 and show that local completeness holds for disjunction $\lor$.

**Question 4. [5 pts]** Write the introduction and elimination rules for disjunction $\lor$ using hypothetical judgments of the form $\Gamma \vdash C \text{ true}$ where $\Gamma$ is a collection of antecedents.
Question 5. [5 pts] Use the hypothetical judgements in Question 4 and state the two structural properties weakening and contraction.

Question 6. [5 pts] Write a proposition that is logically equivalent to \((A \lor B) \supset C\). Your answer should not express trivial logical equivalence based on commutativity, idempotence, or properties of truth and falsehood. Here are examples of bad answers: \((B \lor A) \supset C\), \((A \lor (\top \land B)) \supset C\), and \((A \lor (\bot \lor B)) \supset C\).
Question 7. [5 pts] Suppose that your answer to the previous question is a proposition $D$. Give a proof of $(A \lor B \supset C) \supset D$ true in the natural deduction style.
제 3 절  First-order logic [25pts]

**Question 1.** [5 pts] Write the introduction and elimination rules for the existential quantification using truth judgments of the form $C \text{ true}$. 

**Question 2.** [5 pts] Use the inference rules in Question 1 and show that local soundness holds for the existential quantification.
Question 3. [5 pts] Give the typing rules for proof terms for the existential quantification.

Question 4. [10 pts] Assume the following typing rules for characterizing natural numbers:

\[
\begin{align*}
\text{Nat}_0 & : \text{Nat}(0) \\
\text{Zero} & : \forall x. \text{Nat}(x) \supset \text{Nat}(s(x)) \\
\text{Succ} & : \forall x. \text{Nat}(x) \supset \text{Nat}(s(x)) \\
\text{Eq}_i & : \forall x. \text{Eq}(x, x) \\
\text{Eq}_t & : \forall x. \forall y. \forall z. (\text{Eq}(x, y) \land \text{Eq}(x, z)) \supset \text{Eq}(y, z) \\
\text{Lt}_s & : \forall x. \text{Lt}(x, s(x)) \\
\text{Lt}_n & : \forall x. \forall y. \text{Eq}(x, y) \supset \neg \text{Lt}(x, y)
\end{align*}
\]

Give a proof term of type \( \neg \exists x. \text{Eq}(x, 0) \land \text{Eq}(x, s(0)) \).
제 4 절  Datatypes [40pts]

Question 1. [5 pts] The introduction rules for datatype nat are:

\[
\begin{align*}
0 & \in \text{nat} & \text{natI}_0 \\
\text{t} & \in \text{nat} & \text{natI}_s
\end{align*}
\]

We use \( \text{ind } u(t) \) of \( u(0) \Rightarrow M \mid u(s(x)) \Rightarrow N \) for the proof term corresponding to the elimination rule based on induction on terms. Give its typing rule.

Question 2. [10 pts] We abbreviate \( EQ(m, n) \) as \( m =_{\text{N}} n \) where both \( m \) and \( n \) belong to datatype nat. We use the following proof terms for the introduction rules for \( m =_{\text{N}} n \).

\[
\begin{align*}
\text{eqI}_0 : m =_{\text{N}} n & \Rightarrow 0 =_{\text{N}} 0 \\
\text{eqI}_s(M) : m =_{\text{N}} n & \Rightarrow s(m) =_{\text{N}} s(n)
\end{align*}
\]

Design three proof terms for the elimination rules for \( m =_{\text{N}} n \) which are obtained by inverting the above two introduction rules, and give their typing rules. (Writing only the typing rules is okay.)
Question 3. [5 pts] Give a proof term $eqNat$ of type $\forall x \in \text{nat}. x =_N x$. 
Question 4. [10 pts] Give a proof term $\text{trans}$ of type $\forall x \in \text{nat}. \forall y \in \text{nat}. \forall z \in \text{nat}. x =_N y \implies y =_N z \implies x =_N z$. 
Question 5. [10 pts] Assume the following definition of $plus$:

\[
\begin{align*}
plus & : \text{nat} \to \text{nat} \\
plus & = \lambda x. \lambda y. \text{rec } p(x) \text{ of } p(0) \Rightarrow y \mid p(s(z)) \Rightarrow s(p(z))
\end{align*}
\]

Using definitional equality, give a proof term $comp$ of type $\forall x \in \text{nat}. \forall y \in \text{nat}. x + s(y) =_N s(x + y)$. You may use $eqNat$ given in Question 3.