

Name:

Hemos ID:

CSE-321 Programming Languages 2009 - Quiz 1 Sample Solution

	Problem 1	Problem 2	Total
Score			
Max	50	50	100

1 Structural induction [50 pts]

$\text{mparen} \quad s ::= \epsilon \mid (s) \mid s \ s$

$$\begin{aligned} \text{left}[\epsilon] &= 0 \\ \text{left}[(s)] &= 1 + \text{left}[s] \\ \text{left}[s_1 \ s_2] &= \text{left}[s_1] + \text{left}[s_2] \\ \text{right}[\epsilon] &= 0 \\ \text{right}[(s)] &= 1 + \text{right}[s] \\ \text{right}[s_1 \ s_2] &= \text{right}[s_1] + \text{right}[s_2] \end{aligned}$$

Fill in the blank to complete the proof.

Theorem 1.1. *If $s \in \text{mparen}$, then $\text{left}[s] = \text{right}[s]$.*

Proof. By structural induction on s .

Case $s = \epsilon$:

$$\underline{\text{left}[\epsilon] = 0 = \text{right}[\epsilon]}$$

Case $s = (s')$:

$$\begin{array}{ll} \underline{\text{left}[s'] = \text{right}[s']} & \text{by induction hypothesis on } s' \\ \underline{\text{left}[s] = 1 + \text{left}[s'] = 1 + \text{right}[s'] = \text{right}[s]} & \text{from } \underline{\text{left}[s'] = \text{right}[s']} \end{array}$$

Case $s = s_1 \ s_2$:

$$\begin{array}{ll}
\frac{\text{left}[s_1] = \text{right}[s_1]}{} & \text{by induction hypothesis on } s_1 \\
\frac{\text{left}[s_2] = \text{right}[s_2]}{} & \text{by induction hypothesis on } s_2 \\
\frac{\text{left}[s_1 \ s_2] = \text{left}[s_1] + \text{left}[s_2] = \text{right}[s_1] + \text{right}[s_2] = \text{right}[s_1 \ s_2]}{} & \\
& \text{from } \frac{\text{left}[s_1] = \text{right}[s_1]}{} \text{ and } \frac{\text{left}[s_2] = \text{right}[s_2]}{} \\
& \square
\end{array}$$

2 Rule induction [50 pts]

$$\begin{array}{c}
\frac{}{\epsilon \text{ mparen}} \text{Meps} \quad \frac{s \text{ mparen}}{(s) \text{ mparen}} \text{Mpar} \quad \frac{s_1 \text{ mparen} \quad s_2 \text{ mparen}}{s_1 \ s_2 \text{ mparen}} \text{Mseq} \\
\frac{}{\epsilon \text{ lparen}} \text{Leps} \quad \frac{s_1 \text{ lparen} \quad s_2 \text{ lparen}}{(s_1) \ s_2 \text{ lparen}} \text{Lseq}
\end{array}$$

Fill in the blank to complete the proof.

Theorem 2.1. *If s lparen, then s mparen.*

Proof. By rule induction on the judgment s lparen.

Case $\frac{}{\epsilon \text{ lparen}} \text{Leps}$ where $s = \epsilon$:

$\frac{}{\epsilon \text{ mparen}}$ by the rule *Meps*

Case $\frac{s_1 \text{ lparen} \quad s_2 \text{ lparen}}{(s_1) \ s_2 \text{ lparen}} \text{Lseq}$ where $s = (s_1) \ s_2$:

$\frac{s_1 \text{ mparen}}{} \quad$ by induction hypothesis on s_1 lparen

$\frac{(s_1) \text{ mparen}}{} \quad$ by the rule *Mpar*

$\frac{s_2 \text{ mparen}}{} \quad$ by induction hypothesis on s_2 lparen

$\frac{(s_1) \ s_2 \text{ mparen}}{} \quad$ by the rule *Mseq*

