

Name:

Hemos ID:

CSE-321 Programming Languages 2011 Quiz 6

	Problem 1	Problem 2	Total
Score			
Max	50	50	100

1 Subtyping [50 pts]

Consider the following definition:

$$\begin{array}{ll} \text{type} & A ::= P \mid A \rightarrow A \mid A \times A \mid A + A \mid \text{ref } A \\ \text{base type} & P ::= \text{nat} \mid \text{int} \end{array}$$

We write $A \leq B$ if A is a subtype of B , or equivalently, if B is a *supertype* of A . Complete the following subtyping rules for product types, sum types and function types:

$$A \times B \leq A' \times B'$$

$$A + B \leq A' + B'$$

$$A \rightarrow B \leq A' \rightarrow B'$$

2 Recursive Types [50 pts]

Consider the following definition of simply typed λ -calculus extended with recursive types:

type	$A ::= \text{unit} \mid A \rightarrow A \mid A + A \mid \alpha \mid \mu\alpha.A$
expression	$e ::= x \mid \lambda x:A. e \mid e e \mid () \mid \text{inl}_A e \mid \text{inr}_A e \mid \text{case } e \text{ of } \text{inl } x. e \mid \text{inr } y. e \mid \text{fold}_C e \mid \text{unfold}_C e$
typing context	$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, \alpha \text{ type}$

Question 1. [20 pts] Complete the typing rules for $\text{fold}_C e$ and $\text{unfold}_C e$:

$$\Gamma \vdash \text{fold}_C e :$$

$$\Gamma \vdash \text{unfold}_C e :$$

Question 2. [30 pts] Complete the following reduction rules based on the eager reduction strategy:

$$\text{fold}_C e \mapsto$$

$$\text{unfold}_C e \mapsto$$

$$\text{unfold}_C \text{fold}_C v \mapsto$$
