Simply typed λ-calculus

We use the following fragment of the simply typed λ-calculus. We do not consider base types.

| type       | $A ::= P \mid A \to A$ |
| base type  | $P ::= \cdots$          |
| expression | $e ::= x \mid \lambda x : A . e \mid e \\ e$ |
| value      | $v ::= \lambda x : A . e$ |
| typing context | $\Gamma ::= \cdot \mid \Gamma, x : A$ |

We use the following reduction and typing judgments:

- $e \mapsto e'$ \iff $e$ reduces to $e'$
- $\Gamma \vdash e : A$ \iff expression $e$ has type $A$ under typing context $\Gamma$

\[
\begin{array}{llll}
& x : A \in \Gamma & \vdash x : A & \text{Var} \\
& \Gamma \vdash \lambda x : A . e : A \to B & \text{Lam} & \Gamma \vdash e : A \to B \quad \Gamma \vdash e' : B \\
& e_1 \mapsto e'_1 & e_2 \mapsto e'_2 & \text{App} & (\lambda x : A . e) \mapsto [v / x] e
\end{array}
\]

Question 1. [10 points] State the weakening property of typing judgments:

If $\Gamma \vdash e : C$, then $\Gamma, x : A \vdash e : C$.

Question 2. [10 points] State the progress theorem:

If $\vdash e : A$ for some type $A$,

then either $e$ is a value or there exists $e'$ such that $e \mapsto e'$.

Question 3. [10 points] State the type preservation theorem:

If $\Gamma \vdash e : A$ and $e \mapsto e'$, then $\Gamma \vdash e' : A$.

Question 4. [10 points] State the canonical forms lemma:

If $v$ is a value of type $A \to B$, then $v$ is a λ-abstraction $\lambda x : A . e$.

Question 5. [10 points] State the substitution lemma:

If $\Gamma \vdash e : A$ and $\Gamma, x : A \vdash e' : C$, then $\Gamma \vdash [e/x]e' : C$. 
Question 6. [10 points] State the inversion lemma:

Suppose $\Gamma \vdash e : C$.

If $e = x$, then $x : C \in \Gamma$.

If $e = \lambda x : A.e'$, then $C = A \rightarrow B$ and $\Gamma, x : A \vdash e' : B$ for some type $B$.

If $e = e_1 e_2$, then $\Gamma \vdash e_1 : A \rightarrow C$ and $\Gamma \vdash e_2 : A$ for some type $A$.

Question 7. [10 points] Type safety guarantees that evaluating a well-typed expression (i.e., running a well-typed program) eventually terminates, never producing non-termination. True or false?

False.